

# Extracting Gravitational Waves Induced by Plasma Turbulence in the Early Universe through an Averaging Process

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## Abstract

This work is a follow-up to the paper, "Numerical Relativity as a Tool for Studying the Early Universe". In this article, we determine if gravitational waves can be accurately extracted from a dynamical spacetime using an averaging process as opposed to conventional methods of gravitational wave extraction. We calculate the normalized energy density, strain and degree of polarization of gravitational waves produced by a simulated turbulent plasma similar to what was believed to have existed shortly after the electroweak scale. This calculation is completed using two numerical codes, one which utilizes full General Relativity calculations based on modified BSSN equations while the other utilizes a linearized approximation of General Relativity. Our results show that the spectrum of gravitational waves calculated from the linear code are nearly indistinguishable from those calculated from the nonlinear code using an averaging process. This result validates the use of the averaging process for gravitational wave extraction of cosmological systems.

## I. INTRODUCTION

Recent work by the Bicep 2 Collaboration [1], has resulted in increased interest in the existence of primordial gravitational waves. Although these results were later disproven [2, 3], speculation about the characteristics of primordial gravitational waves is still very much alive [20, 27–34]. Our ultimate goal is to determine the characteristics of these waves, given different cosmological theories, and if possible, find ways to observe them. However, before we can do that we must first determine how to extract gravitational waves produced by turbulence in the relativistic plasma which dominated the universe shortly after the electroweak phase transition. Can it be accurately calculated using linearized approximations of gravity? Does the chaotic nature of the General Relativity’s nonlinearities significantly affect the solution [23]? Can an averaging process be used to quickly extract gravitational waves from a dynamic spacetime [17]?

Primordial plasma turbulence is believed to have occurred as a result of stirring caused by bubble wall collisions and other chaotic events during inflation and the Electroweak phase transition. The characteristic velocity of the turbulent eddies was calculated to be as high as 0.65 [31, 33]. The magnetic fields around this time may have been as high as  $10^{20}$  G. This number was determined by extrapolating the accepted upper limit on cosmic magnetic fields,  $10^{-9}$  G to a much smaller scale factor and assuming that the  $B \propto a^2$  relationship holds [16].

The author’s previous work [18] showed how the framework of numerical relativity could be used to study the cosmology of the early universe. In this article, we expand on the techniques presented and determine the importance of including nonlinear effects. We utilize two General Relativistic Magnetohydrodynamic (GRMHD) codes in order to simulate a turbulent plasma similar to that of the universe when it was less than a second old. One of these codes utilizes full general relativity modeled using a modified version of the BSSN equations in order to simulate the dynamical spacetime background while the other relies on a linearized approximation of gravity. We characterize the gravitational waves produced and then compare the results produced by both codes.

## II. OVERVIEW OF THE SOFTWARE

Our code uses SI units for input and output and geometrized units,  $c = \hbar = G = 1$ , are used for all calculations within the code. Throughout this article, we will refer to time and distance in units of meters.

As described in the preceding paper [18], the codes used here were specifically developed to study early universe cosmology. These codes are based on the Cactus Framework ([www.cactuscode.org](http://www.cactuscode.org)). Cactus was originally developed to perform numerical relativistic simulations of colliding black holes but its modular design has since allowed it to be used for a variety of Physics, Engineering and Computer Science applications. It is currently being maintained by the Center for Computation and Technology at Louisiana State University. Cactus codes are composed of a flesh (which provides the framework) and the thorns (which provide the physics). The codes used within this work, FixedCosmo and SpecCosmo, are a collection of cactus thorns. They are written in a combination of F90, C and C++.

Both codes utilize the relativistic MHD evolution equations proposed by Duez [14]. Both codes are also designed to utilize a variety of different differencing schemes including 2nd order Finite Differencing, 4th order Finite Differencing and Spectral Methods. The key difference between the codes is that while one is capable of solving Einstein's Equations directly (through a modified BSSN formulation) as well as the relativistic MHD equations, the other solves the relativistic MHD equations but simulates an expanding spacetime and estimates the gravitational wave background without solving directly Einstein's Equations. This allows us to perform a test to determine under what conditions it is important to spend computational resources to solve Einstein's equations directly and if the gravitational waves are being correctly extracted from the nonlinear code. The codes were thoroughly tested [18] and found to accurately model known GRMHD dynamics. These tests included MHD waves induced by gravitational waves test, the consistency of cosmological expansion test and shock tests.

The initial data used was derived from work done by several projects involving primordial magnetic fields, phase transitions and early universe cosmology in general [16, 25, 31, 33, 34]. This study models an extremely high energy epoch of the universe shortly after the Electroweak phase transition when the universe was about  $10^{-6}$  s old. The authors chose this as the starting point for our study because it was the beginning of the Hadronic Epoch

of the early universe.

### III. EVOLUTION EQUATIONS

The MHD equations used to evolve both numerical codes were based on Duez's evolution equations [14].

$$\partial_t \rho_* + \partial_j (\rho_* v^j) = 0, \quad (1)$$

$$\partial_t \tilde{\tau} + \partial_i (\alpha^2 \sqrt{\gamma} T^{0i} - \rho_* v^i) = s, \quad (2)$$

$$\partial_t \tilde{S}_i + \partial_j (\alpha \sqrt{\gamma} T_i^j) = \frac{1}{2} \alpha \sqrt{\gamma} T^{\alpha\beta} g_{\alpha\beta,i}, \quad (3)$$

$$\partial_t \tilde{B}^i + \partial_j (v^j \tilde{B}^i - v^i \tilde{B}^j) = 0. \quad (4)$$

Here  $\rho_*$  is conserved density,  $v^j$  is velocity,  $\tilde{\tau}$  is the energy variable,  $\tilde{S}_i$  is the momentum variable,  $s$  is the source term,  $\alpha$  is the lapse term,  $\gamma$  is the determinate of the three metric and  $T^{ij}$  is the stress-energy tensor. The tilde denotes that the term was calculated with respect to the conformal metric.

$$\rho_* = \alpha \sqrt{\gamma} \rho_0 u^0, \quad (5)$$

$$\tilde{\tau} = \alpha^2 \sqrt{\gamma} T^{00} - \rho_* \quad (6)$$

$$\tilde{S}_i = \alpha \sqrt{\gamma} T_i^0, \quad (7)$$

$$s = \alpha \sqrt{\gamma} [(T^{00} \beta^i \beta^j + 2T^{0i} \beta^j + T^{ij}) K_{ij} - (T^{00} \beta^i + T^{0i}) \partial_i \alpha] \quad (8)$$

The first equation comes from conservation of baryon number, the second derives from conservation of energy, the third is conservation of momentum and the fourth is the magnetic induction equation. For this simulation we use Geodesic Slicing,  $\alpha = 1.0$ ,  $\beta_i = 0.0$  for both codes.

The nonlinear code utilizes a first order version of the BSSN equations to simulate the background space-time. For fixed gauge conditions, the modified BSSN equations as defined by Brown [7] are:

$$\bar{\partial}_0 K = \alpha \left( \tilde{A}^{ij} \tilde{A}_{ij} + \frac{1}{3} K^2 \right) + 4\pi\alpha(\rho + S) . \quad (9)$$

$$\bar{\partial}_0 \phi = -\frac{\alpha}{6} K , \quad (10)$$

$$\bar{\partial}_0 \phi_i = -\frac{1}{6} \alpha \bar{D}_i K - \kappa^\phi \mathcal{C}_i , \quad (11)$$

$$\bar{\partial}_0 \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} , \quad (12)$$

$$\begin{aligned} \bar{\partial}_0 \tilde{A}_{ij} = e^{-4\phi} \left[ \alpha (\tilde{R}_{ij} - 8\pi S_{ij}) - 2\alpha \bar{D}_{(i} \phi_{j)} + 4\alpha \phi_i \phi_j + \Delta \tilde{\Gamma}_{ij}^k (2\alpha \phi_k) \right]^{TF} \\ + \alpha K \tilde{A}_{ij} - 2\alpha \tilde{A}_{ik} \tilde{A}_j^k , \end{aligned} \quad (13)$$

$$\bar{\partial}_0 \tilde{\gamma}_{kij} = -2\alpha \bar{D}_k \tilde{A}_{ij} - \kappa^\gamma \mathcal{D}_{kij} , \quad (14)$$

$$\bar{\partial}_0 \tilde{\Lambda}^i = -\frac{4}{3} \alpha \tilde{D}^i K + 2\alpha \left( \Delta \tilde{\Gamma}_{kl}^i \tilde{A}^{kl} + 6\tilde{A}^{ij} \phi_j - 8\pi \tilde{\gamma}^{ij} S_j \right) . \quad (15)$$

The bar denotes a derivative taken with respect to the fiducial metric (defined here to have a determinate of one) and the tilde again denotes a derivative taken with respect to the conformal metric. Also,  $\mathcal{C}_i$  and  $\mathcal{D}_{kij}$  are constraint equations and  $\kappa^\phi$  and  $\kappa^\gamma$  are proportionality constants.  $\rho$ ,  $S$ ,  $S_j$  and  $S_{ij}$  are source terms as found in the standard version of the BSSN equations. Brown et al also defined:

$$\begin{aligned} \mathcal{C}_i &= \phi_i - \bar{D}_i \phi = 0, \\ \mathcal{D}_{kij} &= \tilde{\gamma}_{kij} - \bar{D}_k \tilde{\gamma}_{ij} = 0, \\ \Delta \tilde{\Gamma}_{kl}^i &= \frac{1}{2} \tilde{\gamma}^{ij} (\tilde{\gamma}_{klj} + \tilde{\gamma}_{lkj} - \tilde{\gamma}_{jkl}) , \\ \tilde{R}_{ij} &= -\frac{1}{2} \tilde{\gamma}^{kl} \bar{D}_k \tilde{\gamma}_{lij} + \tilde{\gamma}_{k(i} \bar{D}_{j)} \tilde{\Lambda}^k + \tilde{\gamma}^{\ell m} \Delta \tilde{\Gamma}_{\ell m}^k \Delta \tilde{\Gamma}_{(ij)k} \\ &\quad + \tilde{\gamma}^{kl} [2\Delta \tilde{\Gamma}_{k(i}^m \Delta \tilde{\Gamma}_{j)m\ell} + \Delta \tilde{\Gamma}_{ik}^m \Delta \tilde{\Gamma}_{mj\ell}] , \end{aligned}$$

These equations allow gravitational waves to appear organically from the turbulent plasma field. For the linear code, we approximate the effect of an expanding spacetime and determine the gravitational wave spectrum without utilizing full general relativity. We do this by solving the Friedmann Equations and the linearized gravitational wave equations [25].

$$\begin{aligned} \partial_0 a &= aH \\ \partial_0 H &= -H^2 - \frac{4}{3} \pi (\rho + 3p) \end{aligned}$$

$$\begin{aligned}\partial_0 \tilde{h}_{ij} &= q_{ij} \\ \partial_0 q_{ij} &= \nabla^2 \tilde{h}_{ij} / a^2 - 2(2H^2 + \partial_0 H)(\delta_{ij} \tilde{h}^{ij} - \tilde{h}_{ij}) + 16\pi T_{ij}.\end{aligned}$$

Where  $\tilde{h}_{ij}$  are gravitational perturbations calculated by the linearized code and  $q_{ij}$  are their time derivatives.

#### IV. CALCULATION OF THE GRAVITATIONAL WAVE SPECTRUM AND POLARIZATION

The Gravitational Wave Spectrum is output as characteristic strain, energy density and degree of polarization by the numerical code [37]. For the full GR code, we utilize an averaging process to calculate the gravitational waves produced by the simulation [17]. Here gravitation perturbations can be found by subtracting the mean value of the metric,  $\bar{g}_{ij}$  from it's value at any point and correcting for the scale factor.

$$h_{ij} = (g_{ij} - \bar{g}_{ij}) / a^2 \quad (16)$$

Unfortunately these perturbations are not in the transverse-traceless gauge so the energy density must be calculated as

$$t_{00} = \frac{1}{32\pi G} \langle (\partial_0 h_{ij})(\partial_0 h^{ij}) - \frac{1}{2}(\partial_0 h)(\partial_0 h) - (\partial_\rho h^{\rho\sigma})(\partial_0 h_{0\sigma}) - (\partial_\rho h^{\rho\sigma})(\partial_0 h_{0\sigma}) \rangle. \quad (17)$$

The brackets denote the average over several wavelengths. By utilizing Geodesic Slicing conditions the last two terms are identically zero. The time derivatives of the perturbations can be rewritten in terms of the extrinsic curvature,  $K$ , lapse,  $\alpha$ , Hubble Parameter,  $H = \frac{\dot{a}}{a}$ , and scale factor,  $a$ .

$$\partial_0 h_{ij} \partial_0 h^{ij} - \frac{1}{2} \partial_0 h \partial_0 h = 2[(2\alpha^2 K_{ij} K^{ij} + 4\alpha H K + 6H^2) - (\alpha K + 3H)^2] / a^4 \quad (18)$$

The mass density is then normalized by dividing by the critical density

$$\rho_c = \frac{3H^2}{8\pi G}. \quad (19)$$

This results in a normalized energy density,

$$\Omega_N = -\langle (2\alpha^2 K_{ij} K^{ij} + 4\alpha H K + 6H^2 - (\alpha K + 3H)^2) / (6a^4 H^2) \rangle. \quad (20)$$

The normalized energy density for the linearized code is then simply calculated as

$$\Omega_L = \frac{1}{12H^2 a^4} \langle q_{ij} q^{ij} - \frac{1}{2} (q_i^i)^2 \rangle. \quad (21)$$

For both codes, the magnitude of the characteristic strain is calculated using the quadratic mean of the plus and cross polarizations of the gravitational waves perceived to be traveling along the x, y and z axes. The plus polarizations are calculated by finding the difference between the diagonal transverse perturbation terms while the cross terms relate directly to the perturbations in the off-diagonal transverse terms.

$$h_x^+ = \frac{1}{2} (h_{yy} - h_{zz}) \quad (22)$$

$$h_y^+ = \frac{1}{2} (h_{zz} - h_{xx}) \quad (23)$$

$$h_z^+ = \frac{1}{2} (h_{xx} - h_{yy}) \quad (24)$$

$$h_x^\times = h_{yz} \quad (25)$$

$$h_y^\times = h_{xz} \quad (26)$$

$$h_z^\times = h_{xy} \quad (27)$$

$$h_i(k, t) = \sqrt{|h_i^+(\vec{k}, t)|^2 + |h_i^\times(\vec{k}, t)|^2} \quad (28)$$

$$h(k, t) = \sqrt{h_x(k, t)^2 + h_y(k, t)^2 + h_z(k, t)^2} \quad (29)$$

According to Kahnashvili [30], the degree of polarization can be defined as,

$$P(k, t) = \frac{\langle h^{R*}(\vec{k}, t) h^R(\vec{k}', t) - h^{L*}(\vec{k}, t) h^L(\vec{k}', t) \rangle}{\langle h^{R*}(\vec{k}, t) h^R(\vec{k}', t) + h^{L*}(\vec{k}, t) h^L(\vec{k}', t) \rangle} \quad (30)$$

Where the right and left polarizations for waves traveling along the x, y or z axis are defined as,

$$h_i^R = h_i^+ + i h_i^\times \quad (31)$$

$$h_i^L = h_i^+ - i h_i^\times \quad (32)$$

By expanding the left and right polarizations into plus and cross polarizations, the polarization degree can be rewritten as,

$$P_i(k_i, t) = \frac{2\langle \text{Im}[h_i^+(k_i, t)]\text{Re}[h_i^\times(k_i, t)] - \text{Re}[h_i^+(k_i, t)]\text{Im}[h_i^\times(k_i, t)] \rangle}{\langle |h_i^+(k_i, t)|^2 + |h_i^\times(k_i, t)|^2 \rangle} \quad (33)$$

## V. EXPERIMENTAL SET-UP

Our simulation begins shortly after the electroweak energy scale, at the beginning of the Hadronic Epoch. The numerical values for the initial conditions were calculated using the available literature [25] and are the exact same for both codes. These calculations give us an initial temperature of  $1.30 \times 10^{13}$  K at time  $1.0 \times 10^{-6}$  s. The scale factor and Hubble Parameter are  $a = 2.096 \times 10^{-13}$  and  $H = 7.99 \times 10^6 \text{s}^{-1}$  respectively. The critical mass/energy density at the time was  $1.14 \times 10^{23} \frac{\text{kg}}{\text{m}^3}$ . The characteristic velocity of the turbulent eddies for all runs was set to 0.25 [31, 33]. The magnetic field at the time could have been as large as  $10^{17}$  G so we set the amplitude of the magnetic fields to  $10^{14}$  G for our simulations.

We ran three sets of simulations using the nonlinear and linear codes with the exact same initial conditions. The first simulation utilized a grid 1000 m cubed with  $64^3$  grid points and a timestep of  $10^{-6}$  m. These dimensions were chosen so that any resulting gravitational waves would correspond to the frequency range of Pulsar Timing observations once universal expansion is taken into account. The other two used higher or lower resolutions in order to establish convergence in our results. Geodesic slicing conditions, periodic boundary conditions and Fourier spectral differencing were used for all simulation runs. There were no shocks or discontinuities in the system so we did not utilize our HRSC routines. We also used a 3rd order Iterative Crank Nicolson time scheme for time integration. All runs started with no initial gravitational waves but the density, temperature, velocity and magnetic fields were all perturbed to model that of an early universe space-time [4, 16, 21, 22, 35, 38]. The initial density and temperature were perturbed by random phase cosine functions with an amplitude proportional to their wavenumber,  $k$ , effectively a Fourier series. The initial magnetic field consisted of random phase cosine waves with an amplitude proportional to their wavenumber squared,  $k^2$ . The initial velocity field consisted of random phase cosine waves with an amplitude proportional to their wavenumber cubed,  $k^3$ . Each run utilized 64



TABLE I. Degree of Polarization at the final time.

	X - Direction	Y - Direction	Z - Direction
Nonlinear (Mean)	0.0	-1.56125E-17	0.0
Linear (Mean)	0.0	0.0	0.0
Nonlinear (Standard Deviation)	0.592164099	0.518579627	0.569916566
Linear (Standard Deviation)	0.592333947	0.515687302	0.572444906

processing cores on the Maxwell computing cluster at the University of Houston's Center for Advanced Computing and Data Systems. Over 10,000 iterations were produced for each data run.

## VI. RESULTS

It should be noted that these results have not been extrapolated for present day observations and only represent a relatively short period in the early universe. The strain and normalized energy density outputs for all runs appeared to be independent of  $k$ , so we chose to focus on the mean value of these quantities. As one can see from Figures 1 and 3, the difference between the average strain as calculated by the nonlinear and linear codes is negligible. We see that the same is true for the Energy Density as shown in Figure 2. Based on the data, we can assume that strain and energy density calculations derived from the linear code is accurate to within a part in a thousand of those derived from the nonlinear code.

As shown in Table 1, The degree of polarization for the gravitational waves appears to be the same for both the linear and nonlinear calculations. This calculation was performed from a sample of 5 different times between  $1.0005 \times 10^{-6}$ s and  $1.0006 \times 10^{-6}$ s. As expected, since the initial data was symmetric, there does not appear to be any bias in the degrees of polarizations for the X, Y and Z directions or between left and right polarized gravitational waves generated by the turbulent system. Further, the statistics of the polarizations for both the nonlinear and linear simulations appear to be the same to two significant figures.

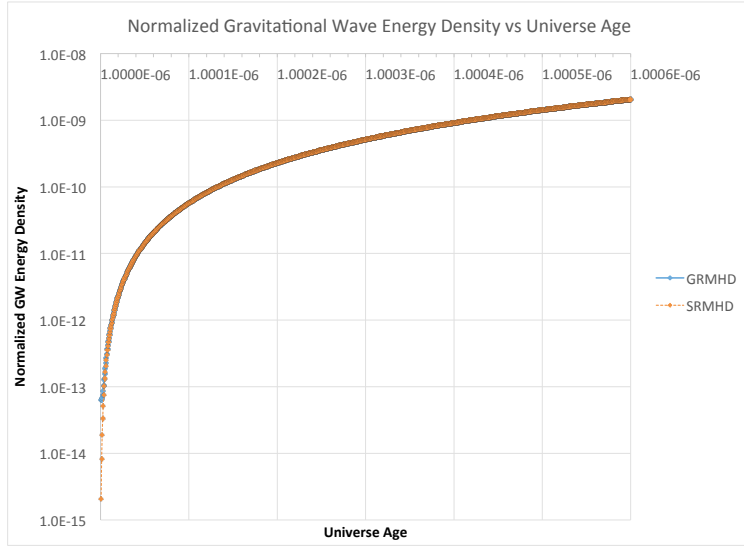
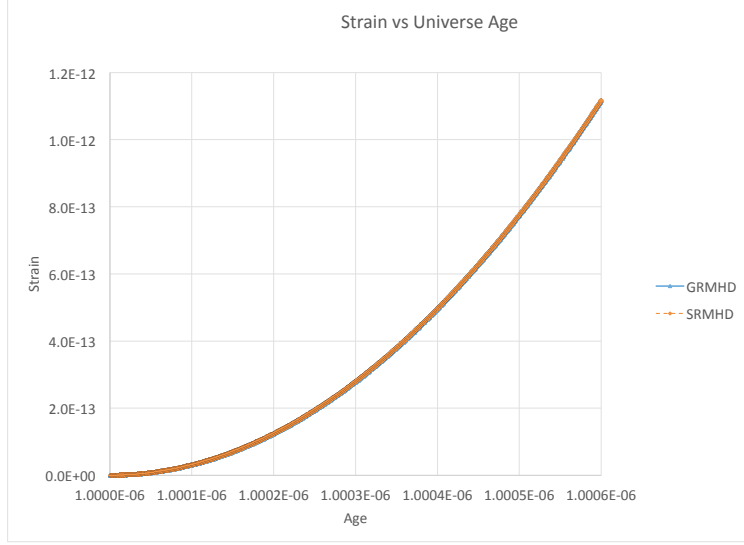


FIG. 1. *Strain of gravitational waves produced by the linear and nonlinear codes. Normalized Energy Density of gravitational waves produced by the linear and nonlinear codes.*

## VII. DISCUSSION

In this article, we tested the averaging process to extract gravitational waves from nonlinear cosmological simulations. We did this by using cosmological linear and nonlinear codes

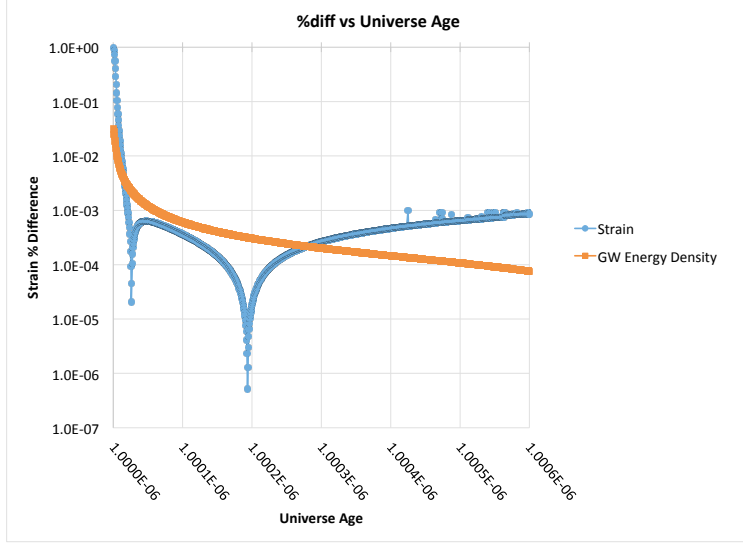


FIG. 2. The %diff was calculated by dividing the difference in strain (energy density) by the strain (energy density) of the nonlinear code.

to solve the exact same problem and comparing the results. We looked at three characteristics of the resulting gravitational waves, strain, normalized energy density and degree of polarization. We found that the results agreed for all three measures. The tiny differences that did occur may be partially explained by small nonlinear effects. We believe that these results prove that the averaging process outlined in Section 4 of this paper is an accurate method of extracting gravitational waves for cosmological systems that lack spacetime singularities. This holds true for gravitational waves with strains up to  $10^{-12}$  and normalized energy densities as high as  $10^{-8}$ . Further work is needed to test the limits of the averaging process. Also, as a result of this work, we can conclude that the linear and nonlinear codes produce the same gravitational waves to within a part in  $10^3$ .

## VIII. CONFLICTS OF INTERESTS

The authors declare that there are no conflict of interests regarding the publication of this article.

## IX. ACKNOWLEDGEMENT

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